Grandma's four-alarm special. In the recipe used thus far for two-generator groups, we restricted the three natural complex parameters $t_a$, $t_b$ and $t_{ab}$ to two by insisting that the trace of the commutator be $-2$. We did this to ensure a reasonable chance that the limit set would be a connected curve. In Box 23 you can see Grandma's prize winning special recipe for arbitrary two-generator groups. In the revised step 2 she let go of the requirement that the three traces satisfy the Markov identity, so the third parameter $t_{ab}$ can also be varied at will.

As you can check, referring back to p. 189 in Chapter 6, the polynomial function of $t_a$, $t_b$ and $t_{ab}$ in step 2 is just the trace of the commutator $C = abAB$. Once we let go of the condition that $\text{Tr}abAB = -2$, there is no reason to expect that the limit set will be anything more than a swirling cloud of dust. This is the recipe to be used to make a picture of a general Schottky group as in our Road Map on the final page.

The special word algorithm revisited

To conclude this chapter, we want to run through the special word algorithm one more time, adapted not to the gasket as we did in Chapter 7, but to the slightly more complicated situation in which the word $aaB$ is parabolic. If we had plotted Figure 8.16 without proper use of this algorithm, there would have been ugly gaps between the parabolic fixed points just like the ones in Figure 7.14. To close the gaps between the pinch points this time, we need to inform the program that the word $aaB$ is parabolic, and plot not only points with repetends.